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Stimulated Brillouin scattering and filamentation of two crossed laser beams in a thermal force dominated plasma

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Abstract : Two crossed laser beams, differing slightly in frequency propagating through a plasma, produces density oscillations at beat frequency via ponderomotive force and nonlinear heating of electrons. The energy exchange between the beams takes place *via* stimulated Brillouin scattering process. If the two beams are of identical frequencies, the process moves over to filamentation instability.

Keywords : Brillouin scattering, filamentation, wave mixing

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1. Introduction

The propagation of nonlinear electromagnetic waves in plasmas is an important area of research. Its understanding is essential for many novel applications *viz.* inertial confinement fusion [1], plasma based accelerators [2], and radiation sources [3]. Majority of the earlier studies were focused on the propagation of a single laser beam in underdense plasmas. These studies identified a number of nonlinear phenomena including self-focusing [4], self phase modulation [5], spot size self modulation [6], hosing [7] *etc.* In recent past, indirect drive approach to inertial confinement fusion (ICF) has been investigated in experiments involving the interaction of a large hohlraum plasma and several laser beams[8]. Basic laser-plasma interaction experiments have investigated the ion acoustic wave spectra that are produced by two crossed beams with a varying intensity ratio. They have confirmed the importance of scattering processes that are simultaneously driven by two beams. A distinctive feature of these interactions is their essentially nonlinear character due to the ion-acoustic waves that are produced in plasmas by the beam interference pattern. These perturbations form a grating and give rise to Bragg diffraction which in turn, enhances forward stimulated Brillouin scattering (SBS) along the propagation directions of the laser pumps. The nonlinear forward SBS leads to rapid time varying energy exchange between beams and further enhances the ion density grating which produces second order Bragg diffraction.

Shukla [9] has studied Brillouin enhanced four-wave mixing and phase conjugation of electromagnetic waves in weakly collisional fully ionized plasma and found that the nonlinearity associated with the nonlocal electron heat transport may dominate over the ponderomotive force and consequently, there appears an enhanced degenerate four-wave mixing and phase conjugation reflectivity. Kruer *et al* [10] have studied scattering between crossing laser beams including the effects of long wavelength modulations in the plasma and thereby energy transfer between them due to induced Brillouin scattering. McKinstrie *et al* [11] have carried out two-dimensional analysis for steady state power transfer between crossed laser beams due to an ion-acoustic wave and this formalism was further extended to three dimensions [12]. Eliseev *et al* [13] have studied parametric interaction of two crossed collimated laser beams with ion plasma modes. Labaune *et al* [14] have observed experimentally nonlinear enhancement of large angle forward scattering of two identical laser beams propagating in preformed plasma due to SBS. Ren *et al* [15] have studied various issues including relativistic effects on the mutual interaction between laser beams in plasmas.

In this paper, we study the nonlinear interaction of two laser beams with frequencies ω_0 and ω_1 to differ slightly in frequency ω in collisional plasma. These pump laser beams exert a static ponderomotive force at $(\omega, \boldsymbol{k}_\perp)$ on the electrons. They also heat the electrons producing a nonuniform temperature at $(\omega, \boldsymbol{k}_\perp)$. This $-\nabla T^4$ ponderomotive force produces electron and ion density perturbations, which couples with $\boldsymbol{v}_0$ to produce a nonlinear current that drives the second beam. The paper is organized as follows: In Section 2 we carry out instability analysis and in Section 3 we discuss our results.

2. Instability analysis

Consider the propagation of two laser beams in an under dense collisional plasma of density n_0^0, electron temperature T_{e0}, and electron- ion collision frequency ν_{ei} (c.f. Figure 1).

$$\boldsymbol{E}_0 = \hat{y}A_0 \exp[-i(\omega_0 t - k_0 z)]$$

and

$$\boldsymbol{E}_1 = \hat{y}A_1 \exp[-i(\omega_1 t - k_{1z} z - k_\perp x)], \tag{1}$$

where $\omega_1 = \omega - \omega_0$, $\boldsymbol{k}_1 = \boldsymbol{k} - \boldsymbol{k}_0$, $\boldsymbol{k} = \boldsymbol{k}_z\hat{z} + \boldsymbol{k}_\perp\hat{x}$,

$$k_{0,1} = \left(\omega_{0,1}^2 - \omega_p^2\right)^{1/2} / c,$$

$\omega_p = \left(4\pi n_0^0 e^2/m\right)^{1/2}$, $-e$ is the electronic charge and m is the electronic mass.

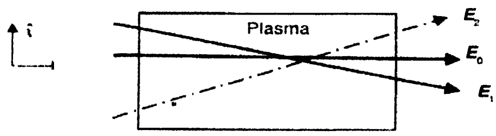

Figure 1. Schematic of the process.

The nonlinear interaction of $\boldsymbol{E}_0$ and $\boldsymbol{E}_1$ produces a laser

$$\boldsymbol{E}_2 = \hat{y}A_2 \exp[-i(\omega_2 t - \boldsymbol{k}_2.\boldsymbol{r})] \tag{2}$$

with the following phase matching conditions

$$\omega_2 = \omega + \omega_0,\ \boldsymbol{k}_2 = \boldsymbol{k} + \boldsymbol{k}_0. \tag{3}$$

The electron dynamics is governed by the equation of motion

$$m(d\boldsymbol{v}_j/dt) = -e\boldsymbol{E}_j - (e/c)\boldsymbol{v}_j \times \boldsymbol{B}_j - m\nu_{ei}\boldsymbol{v}_j, \tag{4}$$

where $\boldsymbol{B}_j = c\boldsymbol{k}_j \times \boldsymbol{E}_j/\omega_j$ being the magnetic fields of the laser and $j = 0, 1, 2$. On solving eq.(4) we obtain the oscillatory electron velocity due to lasers as

$$\boldsymbol{v}_j = e\boldsymbol{E}_j\Big/\left[mi(\omega_j + i\nu_{ei})\right], \tag{5}$$

where $j = 0, 1, 2$.

The component of $\boldsymbol{v}$ in phase with $\boldsymbol{E}$ causes heating at the rate $-(e/2)\,\boldsymbol{E}_j^*.\boldsymbol{v}_j = e^2E_jE_j^*/2m\omega_j^2$. In the steady state, this rate is balanced by the power loss *via* collisions with ions and electrons and thermal conduction *i.e.*,

$$\frac{3}{2}\delta\nu_{ei}T_e = \left(e^2\nu_{ei}E_jE_j^*\right)\Big/\left(2m\omega_j^2\right),$$

where $\delta \sim 10^{-3}$ is the effective fraction of excess electron energy lost in a collision with an ion. It also contains the loss of energy due to thermal conduction. The electron heating rate R due to these lasers can be written as $R = -e(\boldsymbol{E}_0 + \boldsymbol{E}_1 + \boldsymbol{E}_2)(\boldsymbol{v}_0 + \boldsymbol{v}_1 + \boldsymbol{v}_2)$, where the product of the real part of first bracket to the real part of the second bracket is implied.

The $(\omega, \boldsymbol{k})$ component of eq. (6) is

$$R^{(\omega,k)} = -e\left[\boldsymbol{E}_0\cdot\boldsymbol{v}_1 + \boldsymbol{E}_1\cdot\boldsymbol{v}_0 + \boldsymbol{E}_0^*\cdot\boldsymbol{v}_2 + \boldsymbol{E}_2\cdot\boldsymbol{v}_0^*\right]\Big/2 \tag{7}$$

We write the electron temperature T_e as

$$T_e = T_{e0} + T \tag{8}$$

and using in eq.(6), we get the temperature perturbation T as

$$T = \left[\left(\boldsymbol{E}_0.\boldsymbol{v}_1 + \boldsymbol{E}_1.\boldsymbol{v}_0 + \boldsymbol{E}_0^*.\boldsymbol{v}_2 + \boldsymbol{E}_2.\boldsymbol{v}_0^*\right)/2\right]\Big/\left[(3/2)\,(\delta\nu_{ei} + i\omega)\right]$$

$$\approx \left[2e^2\left(\boldsymbol{E}_0.\boldsymbol{E}_1 + \boldsymbol{E}_0^*.\boldsymbol{E}_2\right)\right]\Big/\left[3m\omega^2\delta\,(1 + i\omega/\delta\nu_{ei})\right] \tag{9}$$

here $\nu_{ei} >> \omega$ has been assumed. T goes as $\exp[-i(\omega t - k\ r)]$ $-\nabla T$ can be viewed as a ponderomotive force, hence $-T/e$ can be viewed as a ponderomotive potential

$$\phi_p = -T/e. \tag{10}$$

The electron density perturbation n and ion density perturbation n_i due to ϕ_p and self consistent low frequency potential ϕ are given by

$$n = \left[k^2/(4\pi e)\right]\chi_e(\phi + \phi_p), \tag{11}$$

and

$$n_i = -\left(k^2\phi\right)\Big/(4\pi e) \tag{12}$$

respectively. Here

$$\chi_e \approx \omega_{pi}^2\Big/\left[k^2c_s^2\left(1 - i\nu/k^2v_{th}^2\right)\right],$$

$\chi_i = -\omega_{pi}^2/\left[\omega^2(1 - i\nu m/m_i\omega)\right]$, $\nu \equiv \nu_{ei}$ is the damping rate of the ion acoustic wave, $\omega_{pi} = \sqrt{4\pi n_i Z^2 e^2/m_i}$, $v_{th} = \sqrt{k_B T_e/m}$ $c_s = \sqrt{Zk_B T_e/m_i}$, Z and m_i are ion charge number and mass respectively, k_B is the Boltzmann constant.

Using eqs.(11) and (12) in quasineutrality condition $n \approx n_i$, we get

$$\phi = -(\chi_e/(\chi_e + \chi_i))\,\phi_p. \tag{13}$$

Eq (13) when used in eq. (11), gives

$$n = \frac{k^2}{4\pi e}\frac{\chi_i\chi_e}{(\chi_e + \chi_i)}\phi_p$$

$$\approx -\frac{\omega_\pi^2}{c_s^2 4\pi e}\left[1\Big/\left(1 - \frac{\omega^2}{k^2 c_s^2}\frac{1 + i\nu m/m_i\omega}{1 - i\nu\omega/k^2 v_{th}^2}\right)\right]\frac{T}{e}. \tag{14}$$

n couples with v_0^* and v_0 to produce nonlinear currents J_1^{NL} and J_2^{NL} given by $J_1^{NL} = -nev_0^*/2$ and $J_2^{NL} = -nev_0/2$ respectively. Using these in the wave equation [16]

$$\nabla^2 E_{1,2} + \left[\omega_{1,2}^2 - \omega_p^2\left(1 - i\nu/\omega_{1,2}\right)/c^2\right]E_{1,2}$$

$= -\left[4\pi i\omega_{1,2}/c^2\right]J_{1,2}^{NL}$ we obtain the coupled set of equations for E_1 and E_2 as

$$-2ik_0\frac{\partial E_1}{\partial z} - k^2 E_1 = \frac{\omega_p^2}{2c^2}\frac{|v_0|^2}{v_{th}^2\delta}\frac{(E_1 + E_2)}{(1 + i\omega/\delta\nu)}\frac{1}{D}, \tag{15}$$

and

$$2ik_0\frac{\partial E_2}{\partial z} - k^2 E_2 = \frac{\omega_p^2}{2c^2}\frac{|v_0|^2}{v_{th}^2\delta}\frac{(E_1 + E_2)}{(1 + i\omega/\delta\nu)}\frac{1}{D}, \tag{16}$$

where $D = \left(1 - \dfrac{\omega^2}{k^2 c_s^2}\dfrac{1 + i\nu m/(m_i\omega)}{1 - i\nu\omega/(k^2 v_{th}^2)}\right)$.

If we had taken the pump and the sidebands of the same frequency ($\omega = 0$), then expression for χ_i changes to $\chi_i = \omega_{pi}^2/k^2 v_{thi}^2$ and the above equations for E_1 and E_2 are still valid with $D \to 1$ and $\omega = 0$. This case corresponds to filamentation while the one with $\omega \neq 0$ is Brillouin scattering.

Rearranging eqs.(15) and (16), we get

$$\left[\frac{\partial}{\partial z} + \frac{k^2}{2ik_0} - \frac{\omega_p^2}{2c^2}\frac{1}{2ik_0}\frac{|v_0|^2}{v_{th}^2\delta(1 + i\omega/\delta\nu)D}\right]E_1$$

$$= \frac{\omega_p^2}{2c^2}\frac{1}{2ik_0}\frac{|v_0|^2}{v_{th}^2\delta(1 + i\omega/\delta\nu)D}E_2, \tag{17}$$

and

$$\left[\frac{\partial}{\partial z} - \frac{k^2}{2ik_0} - \frac{\omega_p^2}{2c^2}\frac{1}{2ik_0}\frac{|v_0|^2}{v_{th}^2\delta(1 + i\omega/\delta\nu)D}\right]E_2$$

$$= -\frac{\omega_p^2}{2c^2}\frac{1}{2ik_0}\frac{|v_0|^2}{v_{th}^2\delta(1 + i\omega/\delta\nu)D}E_2. \tag{18}$$

Operating eq. (18) by

$$\left[\frac{\partial}{\partial z} + \frac{k^2}{2ik_0} - \frac{\omega_p^2}{2c^2}\frac{1}{2ik_0}\frac{|v_0|^2}{v_{th}^2\delta(1 + i\omega/\delta\nu)D}\right]$$

and using eq. (17), we get

$$\partial^2 E_2/\partial z^2 - \Gamma^2 E_2 = 0, \tag{19}$$

where

$$\Gamma^2 = \frac{k^2}{4k_0^2}\left[k^2 - \left\{\frac{\omega_p^2}{c^2 v_{th}^2}\frac{1}{D}\frac{|v_0|^2}{\delta(1 + i\omega/\delta\nu)}\right\}\right]. \tag{20}$$

The general solution of eq. (19) can be written as

$$E_2 = A_2 e^{\Gamma z} + A_2 e^{-\Gamma z}. \tag{21}$$

Using the boundary conditions $E_2 = 0$ and $E_1 = E_{10}$ at $z = 0$ or

$$\frac{\partial E_2}{\partial z} = -\frac{\omega_p^2}{2c^2}\frac{1}{2ik_0}\frac{|v_0|^2}{v_{th}^2\delta(1 + i\omega/\delta\nu)D}E_{10},$$

we get

$$A_2 = -A_2 = -\frac{\omega_p^2}{4c^2}\frac{1}{2ik_0\Gamma}\frac{|v_0|^2}{v_{th}^2\delta(1 + i\omega/\delta\nu)}\frac{E_{10}}{D}. \tag{22}$$

Using the conditions $\omega = 0$ and $D \to 1$ in eq.(20), for filamentation instability we obtain the spatial growth rate Γ for the process as

$$\Gamma = \frac{k}{2k_0}\left[k^2 - \left\{\frac{\omega_p^2 v_0^2}{c^2 v_{th}^2\delta}\right\}\right]^{\frac{1}{2}}. \tag{23}$$

In Figure 2 we have plotted the variation of Γ with electron temperature T_e at different values of v_0/c for the following parameters, $\omega_0 = 2\times10^{14}$ rad/s, $n_e = 10^{16}$ cm^{-3}, $\nu_{ei} = 7\times10^{11}$ s^{-1}, the damping rate of ion acoustic frequency $\nu \sim 0.3\times10^8$ s^{-1}. The growth rate shows a saturation behavior at higher temperatures and decreases at higher electron velocities.

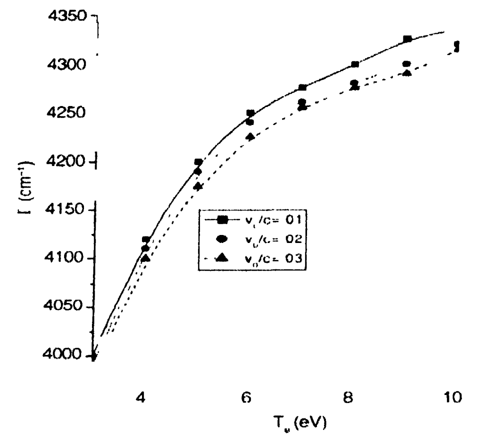

Figure 2. Variation of filamentation growth rate Γ with electron temperature T_e at various values of v_0/c

For $\omega \neq 0$ *i.e.* $(\omega \approx k_\perp c_s)$, from eq. (20), we obtain the SBS growth rate γ by applying basic principles of complex analysis as

$$\gamma = \mathrm{Re}\,\Gamma = \frac{k}{2k_0}\cdot\left(\beta_1^2+\beta_2^2\right)^{\frac{1}{3}}\cos\left[\frac{1}{2}\left(\tan^{-1}\frac{\beta_2}{\beta_1}\right)\right], \quad (24)$$

where

$$\beta_1 = k^2 - \frac{\xi^2\alpha_1}{\alpha_1^2+\alpha_2^2},\ \beta_2 = \frac{\xi^2\alpha_2}{\alpha_1^2+\alpha^2},\ \xi^2 = \frac{\omega_p^2|v_0|^2}{c^2 v_{th}^2\delta},$$

$$\alpha_1 = 1 - \frac{\omega^2/k^2c_s^2}{1+\left(\nu\omega/k^2v_{th}^2\right)^2} + \frac{\nu m}{m_i\omega}\frac{\nu\omega}{k^2v_{th}^2}$$

$$+\left(\frac{\nu m}{m_i\omega}+\frac{\nu\omega}{k^2v_{th}^2}\right)\left[\frac{\omega^2/k^2c_s^2}{1+\left(\nu\omega/k^2v_{th}^2\right)^2}\right]\frac{\omega}{\delta\nu},$$

and

$$\alpha_2 = \frac{\omega}{\delta\nu}\left[1 - \frac{\omega^2/k^2c_s^2}{1+\left(\nu\omega/k^2v_{th}^2\right)^2} + \frac{\nu m}{m_i\omega}\frac{\nu m}{k^2v_{th}^2}\right]$$

$$-\frac{\omega^2/k^2c_s^2}{1+\left(\nu\omega/k^2v_{th}^2\right)^2}\left[\frac{\nu m}{m_i\omega}\frac{\nu\omega}{k^2v_{th}^2}\right].$$

For the following parameters of a nitrogen plasma:

$$v_0/c = 0.01,\quad T_e = 3\text{eV},\quad \omega_0 = 2\times10^{14}\ \text{rad/s}$$
$$n_e = 10^{16}\ \text{cm}^{-3}\quad \nu_{ei} = 7\times10^{11}\ \text{s}^{-1},\quad \nu = 3\times10^7\ \text{s}^{-1},$$

we obtain the SBS growth rate $\gamma = 4.8\times10^3$ cm^{-1}.

3. Discussion

In summary, we have briefly discussed filamentation and SBS of two crossed laser beams in thermal force dominated plasma. The underlying physical mechanism in this interaction is a density grating that is formed due to the interference of crossed laser beams. The two laser beats with each other and produces a density perturbation due to ponderomotive force and electron heating. If two lasers are of equal frequency, we get a static density perturbation which gives rise to filamentation instability. If the two lasers differ slightly in frequency, we get an ion-acoustic grating which causes SBS. The study is useful in energy transfer from one laser to another in X-ray driven inertial fusion where beam smoothing scheme is employed to reduce filamentation and other instabilities, and also in Brillouin enhance four-wave mixing and optical phase conjugation.

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